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A repurposed geometric reasoning model for Engineering Graphics and Design: a conceptual paper

Vernon Candiotes, University of Pretoria, South Africa
Willem Rauscher, University of Pretoria, South Africa
Sonja van Putten, University of Pretoria, South Africa

Abstract
Engineering Graphics and Design is a South African school subject which is foundational to fields such as mechanical, electrical, and civil engineering. However, persistent shortcomings in this subject's instruction have been documented in the South African National Senior Certificate examiners' reports over the past decade. These issues stem primarily from ineffective instructional strategies and a lack of structured reasoning development. These methodological deficiencies may influence the development of learners' visuospatial reasoning skills and conceptual understanding. The situation is exacerbated by the fact that no model for developing such conceptual understanding and reasoning currently exists in the Engineering Graphics and Design field. The need for such a model prompted our search for a suitable model for the structured development of visuospatial reasoning skills, culminating in this conceptual paper. We address the gap by demonstrating how the van Hiele model of geometric reasoning can be particularised to suit the needs for the development of visuospatial reasoning skills in this subject. Our methodology involved the extraction of nine cognitive descriptors from the relevant literature dealing with the van Hiele model. We explain how these descriptors align with Engineering Graphics and Design reasoning requirements. The proposed model offers both diagnostic capabilities for assessing student reasoning levels and instructional guidance for systematic skill development. It is recommended that empirical studies be conducted to test the usability of this repurposed model both in teaching and in the evaluation of the levels of reasoning in assessments.

Keywords
Engineering Graphics and Design; van Hiele model for geometric reasoning; visuospatial reasoning; orthographic and isometric drawing

Background
Engineering Graphics and Design (EGD) is an elective subject offered to Grades 10- 12 learners in South African schools. In this subject, the language of the field is introduced complete with its own operational symbols, lines, and grammatical rules (Bertoline et al., 2011). This language enables verbal and non-verbal communication, allowing technical ideas to be conveyed with clarity. For many EGD learners, essential reasoning skills are poorly developed or lacking (Khoza, 2014; Ramatsetse et al., 2023; Sotsaka & Singh-Pillay, 2020). Information obtained from the South African National Senior Certificate (NSC) examiners' reports for EGD, indicates that Grade 12 learners make certain persistent errors (Department of Basic Education, 2021, 2022). These reports include the following: the inability to read and interpret engineering drawings; difficulty in analysing and planning;

displaying poor knowledge of EGD conventions; poor analysis of question data and requirements; poor planning; poor transfer of Grades 10 and 11 knowledge; inability to calculate simple volumes and areas; misconceptions with orthographic projections; inability to construct and rotate solid geometry figures; inability to convert orthographic multiviews to isometric views; poor execution of assembly drawings; incorrect hatching of sectioned views; and poor visualisation abilities in navigating between 2D and 3D spaces. The teaching and learning of visuospatial reasoning (VSR) is a thread which lies across most of the problematic areas that have been highlighted in these examiners' reports of the last 11 years.

VSR is also referred to as spatial intelligence, spatial thinking, spatial cognition and spatial expertise (Hegarty, 2010). The domain of spatial reasoning ability is contentious as much research has been conducted on it, yet defined attributes appear fragmented, and a single definition does not yet exist (Seery et al., 2015). Newcombe and Shipley (2014, p. 2) argue against the notion of a unitary approach to VSR and state that, "Sadly, the truth is that a hundred years or so of work with existing tests and statistical techniques has not arrived at a cohesive view of the structure of spatial intellect".

The 2021 Department of Basic Education report alludes to problems with teacher competence, as evidenced by an analysis of Grade 12 final examinations. It may well be that the problems lie within the schooling system itself. Hurrell (2021) contends that schooling systems that focus more on procedural knowledge than conceptual knowledge are prone to encourage superficial learning, which negatively impacts the transfer of learning.

Research by Makgato (2016) on preservice EGD teachers found that they are generally underprepared in terms of their VSR ability. Potter et al. (2009) conducted longitudinal studies on EGD over a period of two decades and found that most first-year university students experienced difficulties with VSR.

A similar situation exists in other parts of the world where preservice teachers were found not to know the basic concepts well enough to understand complex concepts and to teach at the correct level (Cunningham & Roberts, 2010). Globally, spatial training is historically missing from most curricula (Taylor & Hutton, 2013; Uttal et al., 2013; Potter et al., 2009). As an unintended consequence, many students in STEM fields who display low levels of spatial ability are dropping out of tertiary education (de Rosa & Fontaine, 2018). As a compounding factor, the White Paper on Post-school Education and Training presented by the Department of Higher Education and Training (2013) hints at current lecturer shortcomings, especially in STEM subjects.

Although considerable progress has been made to increase enrolment in STEM fields, drop-out rates are undermining attempts to retain and grow student numbers in such programmes (Pinxten et al., 2015). To curtail the drop-out trend in STEM fields, it is vital to employ strategies for the early detection of at-risk students regarding spatial ability (Potter et al., 2009; Khoza, 2014; Pinxten et al., 2015). Battista (2007, 2011) argues for the need to understand students' thought processes in order to provide them with a meaningful education. It is essential to identify the cognitive processes that underlie geometry processes in learners in order to determine the nature of the difficulty they are experiencing (Duval, 2006; Lithner, 2000; Battista, 2007; Makina & Wessels, 2009). Scaffolding the development of students' spatial abilities has become a crucial concern in spatial ability research (Buckley, 2018; Luh & Chen, 2013; Gal & Linchevski, 2010; Uttal et al., 2013). However, Taylor and Hutton (2013) argue that efforts towards developing spatial ability should focus on the teaching process rather than modified or additional content.

Conceptual knowledge in EGD involves VSR skills. Research findings suggest that it is common for teachers of EGD to lack the teaching skills required for learners to develop such skills (Khoza, 2014; Perez Carrion & Serrano, 2012; Singh-Pillay & Sotsaka, 2016). Teachers are often unable to master certain spatial and geometric concepts themselves and may therefore, by default, impart the same misconceptions to their learners (Kell et al., 2013; Khoza, 2017; Verdine et al., 2017). In addition, EGD teachers often experience difficulties with certain subject content and lack the understanding to plan effective teaching and learning experiences (Khoza, 2017; Marunić & Glazar, 2014). Documented research shows that teachers' own spatial skills influence their pedagogical choices: weaker spatial abilities lead to reduced use of spatial tools like sketching, manipulatives, diagrams, and gestures in teaching, and are often accompanied by increased spatial anxiety, particularly with regard to the manipulation of 2D and 3D objects that may limit students' learning opportunities (Atit & Rocha, 2021).

Interest in the efficacy of pedagogical practices in Science, Technology, Engineering, and Mathematics (STEM) education is on the increase due to the significant correlation between STEM students’ performance and spatial reasoning skills (Carbonell Carrera et al. 2011; Cheng & Mix, 2014; Harle & Towns, 2011; Marunić & Glazar, 2014; Sorby, 2009; Uttal & Cohen, 2012). Studies by Delahunty et al. (2020) and Wai et al. (2009) on the conceptualisation of STEM problems confirm Tversky’s (2005) belief that advanced spatial skills utilise VSR processes that generate substantial mental representations of problems for successful problem solving. Delahunty et al. (2016) caution that the etiological relationship between STEM success and spatial reasoning ability is not yet well understood. Nevertheless, McLain (2022) states that the unique pedagogical approaches and instructional strategies of Design and Technology education can be of value to a broad and balanced curriculum.

As a subject, Engineering Graphics and Design is not the sole proprietor of spatial reasoning: substantial research links spatial reasoning skills to mathematics and STEM teaching and learning across grades. Lowrie and colleagues conducted large-scale intervention studies demonstrating that spatial training programs significantly improve both spatial reasoning and mathematics performance in primary school students (Lowrie, Logan, & Ramful, 2016; Lowrie, Logan, Harris, & Hegarty, 2018). Importantly, these improvements extended beyond geometry and measurement tasks to include number and algebra problems, suggesting that the development of spatial reasoning skills supports a wider range of mathematical thinking. There is therefore a sound argument based on empirical support for the development of explicit instructional frameworks in spatially demanding disciplines like Engineering Graphics and Design.

Teachers' spatial abilities significantly influence instructional approaches. Nationally representative data reveal substantial variation in spatial skills across teacher types: secondary STEM teachers possess spatial skills 0.8 standard deviations higher than preschool and primary teachers, with differences persisting after controlling for intelligence and gender (Atit, Miller, Newcombe, & Uttal, 2018). Compared to the general population, 79% of secondary STEM teachers demonstrated above-average spatial skills, versus only 47% of preschool and primary teachers. These disparities are concerning given that spatial skills are fundamental to STEM learning from early educational levels (Atit & Rocha, 2021). These findings have direct implications for Engineering Graphics and Design teacher preparation. Professional development addressing teachers' own spatial reasoning, combined with pedagogical frameworks supporting systematic spatial instruction, may be essential for ensuring effective instruction. The adapted van Hiele model provides such a framework, enabling teachers to diagnose students' conceptual understanding, identify misconceptions, and design appropriately sequenced instruction.

The design of effective instructional systems must be grounded in a clear understanding of the interrelatedness of cognitive processes that support effective VSR (National Research Council, 2006). The interrelatedness of spatial factors and their numerous implications for teaching and learning in STEM places a responsibility on teachers to understand VSR on a deeper level (Delahunty et al., 2016). Several researchers mention a gap between understanding the complex cognitive involvement with VSR tasks and the absence of a tailormade framework to classify the different cognitive processes (Buckley & Seery, 2016; Newcombe & Shipley, 2014; Pittalis & Christou, 2010; Seery et al., 2015).

But what is VSR actually? Sir Francis Galton (1883) is credited for the conceptualisation of the construct of spatial reasoning ability using mental imagery studies as early as 1880 (Mohler, 2008). In Thorndike's (1921) work, "mechanical intelligence" was defined as the ability to visualise relationships among objects and to comprehend various functions of the physical world. Thurstone (1938) suggested a "space" factor as a mental ability to process spatial and/or visual images. He was among the first to propose that intelligence is not a singular factor and demonstrated his ideas through his Multiple Factors theory. Thurstone (1950) used abstract terms to propose three factors central to spatiality, S1, S2 and S3. Mental rotation (S1) is the ability to maintain recognition of objects when a change in their orientation occurs. Spatial visualisation (S2) is defined as the ability to recognise the components of objects being displaced from their original position. Spatial perception (S3) relates to spatial orientation by manipulating one's own body orientation. These definitions were subsequently replaced with terms that most people could relate to. In general terms, spatial visualisation is the capacity to visualise objects in space and to be perceptive of the internal and external relationships which allow them to be transformed and manipulated. Spatial reasoning skills are mental functions to reason about spatial relationships in imagined and real spaces and manipulate and organise mental images (Newcombe & Shipley, 2014; Uttal et al., 2013).

Hendel (2021) posits that hierarchical teaching and learning models such as those devised by Bloom et al (1956), Gagne (1985), Van Hiele (1986), Anderson and Krathwohl (2001) and Marzano (2001), have led to educational improvement across the globe. However, a tailormade framework for geometric reasoning in EGD has remained elusive. The literature in this regard points to several obstacles in STEM education and, in particular, to EGD, where research is needed to provide effective teaching and learning. The purpose of this article is to demonstrate how the framework gap can be filled by particularising a version of the van Hiele model for geometric reasoning for use in EGD. Therefore, the question that would guide our research was framed as follows: how can a proven framework for geometric reasoning skills be repurposed for use in EGD?

Understanding concepts in Engineering Graphics and Design

In educational research, concepts represent organising principles that structure understanding within a domain, distinct from procedural knowledge. In mathematics, concepts such as "equivalence" or "function" represent fundamental organising ideas, whereas procedures represent specific solution steps. Similarly, in science, concepts like "energy transformation" provide frameworks for organising and interpreting phenomena. Thus, conceptual understanding involves the "why" of an organisational principle, whereas procedural knowledge refers to the "how" or the steps in implementing such a principle.

Within Engineering Graphics and Design, concepts represent fundamental principles underpinning spatial reasoning, projection systems, and technical communication. Nabutola et al. (2018) identified ten fundamental concepts through multi-institutional collaboration: Visualising in 2D and 3D, Mapping between 2D and 3D, Planar Graphical Elements, Sectional Views, Methodologies for Object Representation, Projection Theory, Parallel Projection Methodologies, Drawing Conventions, Dimensioning, and Solid Modelling Constructs.

Consider projection theory as an example. This concept deals with the principles governing how three-dimensional objects are systematically represented in two dimensions, not merely procedural steps for creating views. Conceptual understanding includes grasping geometric relationships between projection planes, understanding *why* views align in specific configurations, and recognising how dimensional information transfers between views. Students with strong conceptual understanding can explain *why* projections work and they can flexibly apply principles to a variety of contexts, whereas procedural knowledge enables correct execution, i.e. *how* to do it, without understanding underlying geometric principles.

The distinction between conceptual and procedural knowledge has significant instructional implications. Nabutola et al. (2018) prove that building on poor conceptual foundations leads to misconceptions and academic problems. Misconceptions—systematic errors in understanding, rather than random errors— persist and compound as students encounter more complex material. Students commonly exhibit specific misconceptions about core concepts, like view orientation, projection layout conventions, or alignment relationships (Nabutola et al., 2018).

The van Hiele model adaptation provides a developmental framework for conceptual progression in geometric reasoning. Just as the original model describes how geometric thinking develops through increasingly sophisticated conceptual levels, the adapted model provides teachers with structured approaches for supporting students' progression from informal spatial intuitions through formal conceptual frameworks.

Filling the framework gap

Delahunty et al. (2016) state that the interrelatedness of spatial reasoning ability factors has numerous implications for STEM education. Yet, two years later, when Buckley (2018) conducted intensive research on the factors that encompass our understanding of spatial reasoning ability, he found that a single, suitable framework still did not exist. Both STEM teachers and students generally find VSR to be problematic (Clark & Ernst, 2012; McLaren, 2008; Metraglia et al., 2015; Ruckpaul et al., 2015). Frameworks specific to the effective teaching and learning of EGD with VSR as the central construct are lacking.

In our search for an appropriate theoretical framework, we considered the Structure of Observed Learning Outcomes (SOLO) of Biggs and Collis (1982), the Six-Stage Theory of Dienes (1960) and the Actions, Processes, Objects, and Schemas (APOS) theory (Dubinsky, & McDonald, 2001). Biggs and Collis (1982) created the SOLO model which facilitates the teaching and learning of geometry through a structured framework of assessment of learners' geometry insight at different levels. The SOLO model advocates progression from surface-level understanding (identification of shapes and their basic properties) to deeper, more complex reasoning (analysis of relationships for applying existing schemas in new contexts), without defining cognitive milestones.

The Six-Stage Theory of Dienes (1960) focuses on the progression of thinking using concrete manipulatives which gradually makes way for abstract thinking. This theory lacks a structure of hierarchical cognitive levels of thinking and does not provide learning/teaching phases.

The APOS Theory (Dubinsky & McDonald, 2001) focuses on learners' mental structures of action, reasoning processes, and schemas to explain how mathematical concepts are understood. In learning geometry, learners progress from performing actions such as constructing shapes to generating schemas for complex geometric relationships. However, APOS is more appropriate for developing general mathematical concepts rather than geometry-specific thinking.

In contrast, the van Hiele model does provide a hierarchical cognitive structure to describe the development of geometry-specific thinking, thus remedying what may be seen as shortcomings in the models we discuss above. In this regard, Braithwaite (2022) argues that the van Hiele model may be immutable in terms of other, non-geometry mathematical areas, which, in turn, may be conducive to cognitive rigidity. Braithwaite (2022) emphasises the dependence in geometric proof on such skills as visualisation, which he deemed to be less useful in other mathematical areas. However, the van Hiele model has proven its usefulness in geometry education by virtue of its structured hierarchical framework, in which cognitive skills are acquired through a scaffolded teaching and learning process (Karakuş & Peker, 2015).

The van Hiele model characterises students' learning of geometry on hierarchical levels of reasoning (Fuys et al., 1988). The model also proposes five learning phases to improve students' acquisition of reasoning levels by arranging the learning environment according to their prior learning and ability to acquire new levels of reasoning (Karakuş & Peker, 2015). Content that builds onto previous-level cognitive acquisitions can be strategically integrated into the learner's prior body of knowledge by taking cognisance of foundational concepts that should already be acquired. Most studies based on the van Hiele model have only considered the learner-centred part of the model that focuses on cognitive diagnostics (Slavin, 2018).

Guided by Slavin's (2018) thoughts on van Hiele's five learning phases, it is our contention that the teacher-centred part of the van Hiele model is just as important, and efforts to focus purely on diagnostics will not necessarily lead to practices of improved teaching and learning. The van Hieles developed the hierarchical levels to separate different cognitive skills and to demonstrate how instructional strategies should be designed for learners to progress through the levels. The process allows students to cycle and re-cycle through the learning phases to address the non-acquired conceptual skills until they achieve competence.

The five instructional/learning phases suggested by the van Hiele model should incorporate the following cognition areas through the hierarchy of cognitive levels of reasoning:

1. Recognise and differentiate between 2D shapes and 3D objects according to their order of governing properties.
2. Use the principles of both informal and formal inductive and deductive reasoning to relate figure properties within same objects and across different 2D and 3D views of the same object.
3. Identify and distinguish between the necessary and sufficient conditions for a concept to form meaningful definitions towards formal arguments to justify a reasoning path.
4. Apply critical, logical reasoning through theorems, axioms, and definitions in the context of EGD's axiomatic systems.
5. Apply logical reasoning in structuring figure properties and manipulating intrinsic characteristics of relations to derive further information from given data (transition pieces with branches of interpenetration require such reasoning).
6. Acquire technical language through which the properties of concepts can be described.
7. Forge new relationships between concepts while maintaining, refining and renewing existing concepts. For students to progress through the levels of reasoning, conceptual understanding of such re-arrangements must occur.

The van Hiele model has not been particularised for EGD or any other STEM fields besides mathematics. Usiskin (1982) states that cognitive development in geometry can be accelerated through the van Hiele levels by purposeful instruction, exploration, and reflection. Jaime and Gutierrez (1995) noted, "the van Hiele model of mathematical reasoning has become a proven descriptor of the progress of learners' reasoning in geometry and is a valid framework for the design of teaching sequences in school geometry" (p. 592). Pittalis and Christou (2010) argue that not enough research has been conducted to understand VSR phenomena from an integrated visuospatial and geometry perspective in subjects where learners have to switch mentally between 2D and 3D transformation of objects. EGD is such a subject.

The appropriateness of adapting the van Hiele model for Engineering Graphics and Design is further supported by research demonstrating that spatial and geometric understanding develops through hierarchical levels of increasingly sophisticated conceptual frameworks. Lowrie and colleagues' implementation of the ELPSA (Experience-Language-Pictorial-Symbolic-Application) pedagogical framework for spatial reasoning instruction is similar to the van Hiele model, with both frameworks recognizing that learners progress from concrete, experience-based understanding through increasingly abstract and formalised reasoning (Lowrie & Patahuddin, 2015; Lowrie et al., 2018). The ELPSA framework begins with experiential, hands-on engagement, progresses through language development and pictorial representation, advances to symbolic manipulation, and culminates in flexible application—a developmental sequence that parallels the van Hiele levels' progression from visual recognition through informal and formal description to relational reasoning. Similarly, research on engineering graphics concept development demonstrates that students' understanding progresses from recognition of surface features through increasingly sophisticated grasp of underlying geometric and projection principles (Nabutola et al., 2018), further supporting the applicability of hierarchical models like van Hiele to engineering graphics domains.

The van Hiele model of geometric reasoning

A husband-and-wife team from the Netherlands, Pierre van Hiele and Dina van Hiele–Geldof postulated a theory in 1957 for the teaching and learning of geometry, based on their research on how learners progress through learning geometry concepts. In the 1960s, education specialists in the Soviet Union redesigned the geometry national curriculum. The results were so good that they caught the eye of American researchers like Usiskin (1982) and Senk (1985). Their work changed the teaching of geometry on an international scale.

Pierre and Dina van Hiele believed that most of the problems experienced by learners are situated in instructional practices rather than in the cognitive processes utilised for geometry reasoning (Pegg, 2020). The model consists of five hierarchical levels of cognitive reasoning (Pegg & Davey, 1998), which they numbered from 0 to 4. The American version of the model has the levels labelled from 1 to 5. Pierre van Hiele stated that "the levels are situated not in the subject matter but in the thinking of man" (Van Hiele, 1986, p. 41). Henceforth, the levels are referred to as VH1 through to VH5. Each level is defined by cognitive descriptors of how learners reason as they progress in a linear order from Level 1 to Level 5. Figure 1 represents the hierarchical structure of the van Hiele model's five cognitive levels of reasoning.

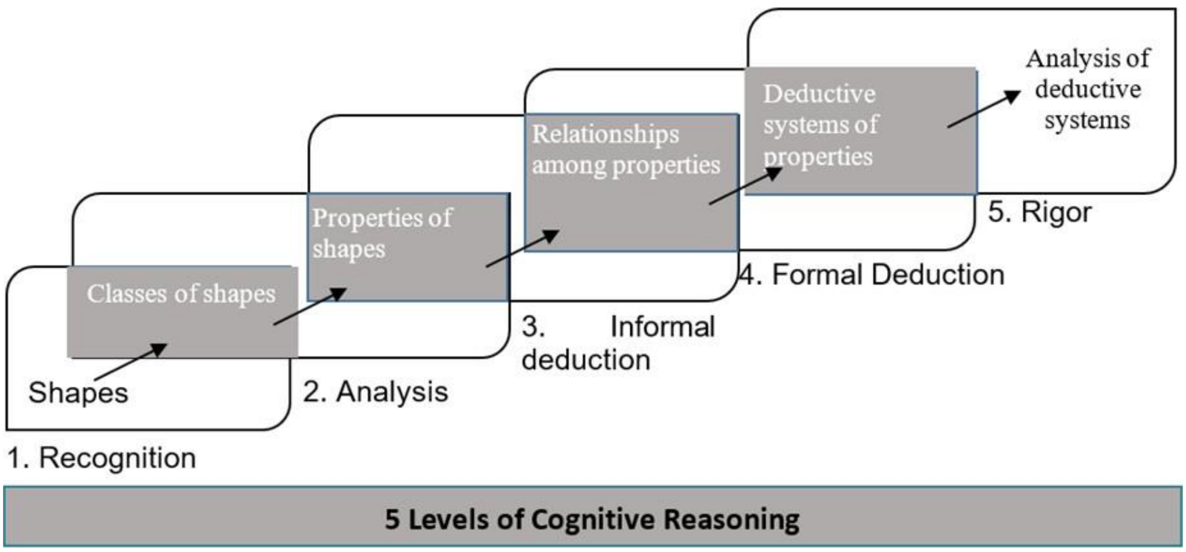

Figure 1: Van Hiele's five levels of cognitive reasoning

Five levels of cognitive reasoning

VH1: Recognition

Figures are visually recognised by their global characteristics (Abdullah & Zakaria, 2013; Gutierrez, 1992). Figures such as squares, triangles and parallelograms are recognised according to their shape, but learners are not yet able to identify figure-properties explicitly (Burger & Shaughnessy, 1986). This level is characterised by learners' ability to observe shapes, take them apart, and rebuild or manipulate them somehow. The focus is on the classification of shapes by exploring their similarities and differences, physically and mentally (van de Walle et al., 2019). In the area of EGD, competency on this level is an assumption because shape recognition is foundational to the field. Since reasoning on Level 1 is intrinsic to visuospatial reasoning of any kind, this level is not included in our application of this model to EGD.

VH2: Analysis

Learners start to consider classing shapes by analysing the properties of figures and learning to describe them appropriately by their technical terminology but are still unable to interrelate the properties within a figure or a figure with other figures (Haviger & Vojkůvková, 2015; Watan & Sugiman, 2018). Learners focus on one class of shapes and can reason that rectangles are rectangles because they have four sides, with opposites being parallel and four 90° angles, their opposite sides equal lengths, and diagonals being equal in length (Curran, 2015). Size and orientation are not considered at this level. Learners begin to grasp those shapes that belong to a certain class, such as cubes, and share corresponding properties. For example, the six congruent faces of cubes are all squares (van de Walle et al., 2019).

VH3: Informal deduction

Learners move beyond just considering figure properties, but also the sufficiency of necessary conditions that allow them to find the relationships between properties of specific shapes (Curran, 2015; Feza & Webb, 2005). Learners apply short deduction steps to arrange the properties of figures logically, and they can grasp such concepts as class inclusions and other interrelationships between figures (de Villiers, 2010; Gutierrez, 1992). Many learners experience difficulty with geometry because of their inability to move beyond VH3 (Haviger & Vojkůvková, 2015). When learners no longer focus only on one particular shape and start to identify other object properties, relationships between various properties of an object and related objects start to make sense. According to van de Walle et al. (2019, p.505), "If all four angles are right angles, the shape must be a rectangle. If it is a square, all angles are right angles. If it is a square, it must be a rectangle." When learners start to engage in if-then reasoning, they are able to use a minimum set of defining attributes to classify shapes. The ability to engage in informal logical reasoning is a signature attribute of Level 3 reasoning. Because they comprehend various properties of shapes, they can ask "Why?" or "What if?" (Pusey, 2003).

VH4: Formal deduction

Learners on this level display a high level of VSR (Gutierrez, 1992). Learners start to comprehend the significance of deductive reasoning, as evidenced by the development of longer sequential statements and start to function with axioms, proofs and theorems. Learners can now analyse informal arguments and the structures of systems inclusive of their axioms. Geometric truth emerges as they begin to use definitions, theorems and corollaries (Ndlovu & Mji, 2012; van de Walle, et al., 2019). Learners can now reason beyond just the properties and utilise networks of relations to derive further information from information that was given through logical thinking rather than intuition most of the time (van Putten, 2008).

VH5: Rigor

Learners move beyond reasoning within one axiomatic system and start to compare and contrast various axiomatic systems. On the most advanced level of the van Hiele hierarchy, axiomatic systems are no longer just the deductions within a system, but the actual axiomatic system becomes the focal point of interest (Frazee, 2018). Researchers such as Usiskin (1982) state that Level 5 cannot be tested as it provides theoretical value only and falls outside the ambit of school geometry. For this reason, Level 5 is excluded from our application of this model to EGD.

The model's strength lies in its hierarchical structure, where each level builds upon previous ones, and its dual focus on diagnosis and instruction. Students must master lower levels before progressing to higher ones, though development is not necessarily linear (Patkin & Barkai, 2014).

Demonstration of the particularisation process

As an example of this process, we selected a simple machine casting which is represented in Figure 2. In this figure, three 2D orthographic multiviews (front, right and top views, on the lefthand side of the figure) and a 3D isometric view are presented on the right in Figure 2. In this example the three orthographic multiviews are used to create the 3D isometric view.

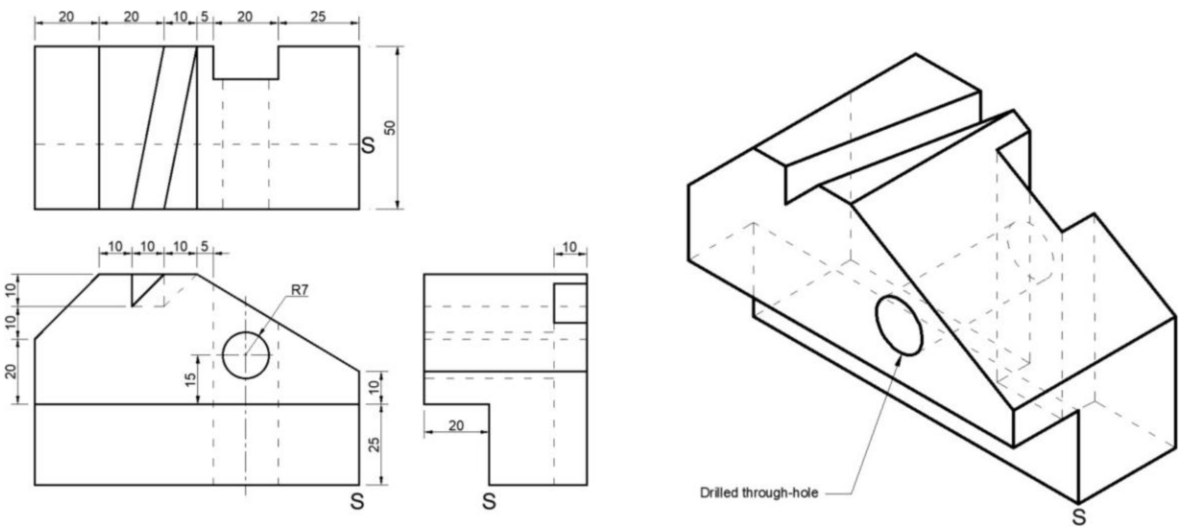

Figure 2: Multiviews and Isometric view of a machine casting

This exercise involves only van Hiele Levels 2, 3, and 4. According to van der Sandt (2007), learners should be functional on Level 3 when they exit the primary school system. As explained in our description of Level 1 earlier, we have excluded the reasoning type of van Hiele Level 1 because the cognitive descriptors of VH2, VH3, and VH4 are more suitable for demonstrating the relationship between EGD and van Hiele. Level 5 is also excluded as it falls outside the ambit of school curricula.

We conducted a wide search of the literature on van Hiele's model of geometric reasoning and extracted nine cognitive descriptors that were found to be appropriate according to our experience with EGD reasoning. These nine cognitive descriptors are listed in Table 1. We identified three cognitive descriptors on VH 2 (Analysis) which align with the reasoning in creating the isometric view shown in Figure 2. On VH3 (Informal deduction), we identified an additional three cognitive descriptors which apply to reasoning associated with creating an isometric view from three orthographic multiviews. Three cognitive descriptors were identified on VH4 which aligns with associating 2D views with corresponding 3D views.

Table 1: Van Hiele cognitive descriptors for geometric reasoning

Van Hiele descriptors for geometry
VH2: Analysis
Can differentiate between types of shapes (Van de Walle, et al., 2019). Classify types of shapes according to governing properties (Patkin & Barkai, (2014). Reasoning is mostly inductive (Curran, 2015).
VH3: Informal deduction
Recognise the importance of properties, and the relationships between them. Can recognise a square as also being a rectangle by definition (De Villiers, 2010). Can order geometric properties and connect them deductively through logical arguments (Karakuş & Peker, 2015). Learners can distinguish between necessary and sufficient conditions for a concept. They can form meaningful definitions and give informal arguments to justify their reasoning (Bleeker et al., 2013).
VH4: Formal deduction
Grasps the significance of deduction. Can reason formally within the context of a mathematical system (axiomatic), complete with undefined terms, axioms, and underlying logical systems with definitions and theorems (Pegg, 2020). Properties are structured to derive further information from given data. Uses logic more than intuition (Burger, & Shaughnessy, 1986). Manipulate intrinsic characteristics of relations (Haviger & Vojkůvková, 2015).

Using Figure 3, we demonstrate how the nine cognitive descriptors of Table 1 pertain to the reasoning types when an isometric view is created from the orthographic views depicted in Figure 3. Some net-shapes were coded in shades of grey on both the orthographic views and the isometric view to show how net-shapes align across different views. Alphabetical characters were used to trace coordinates of the same type across different views.

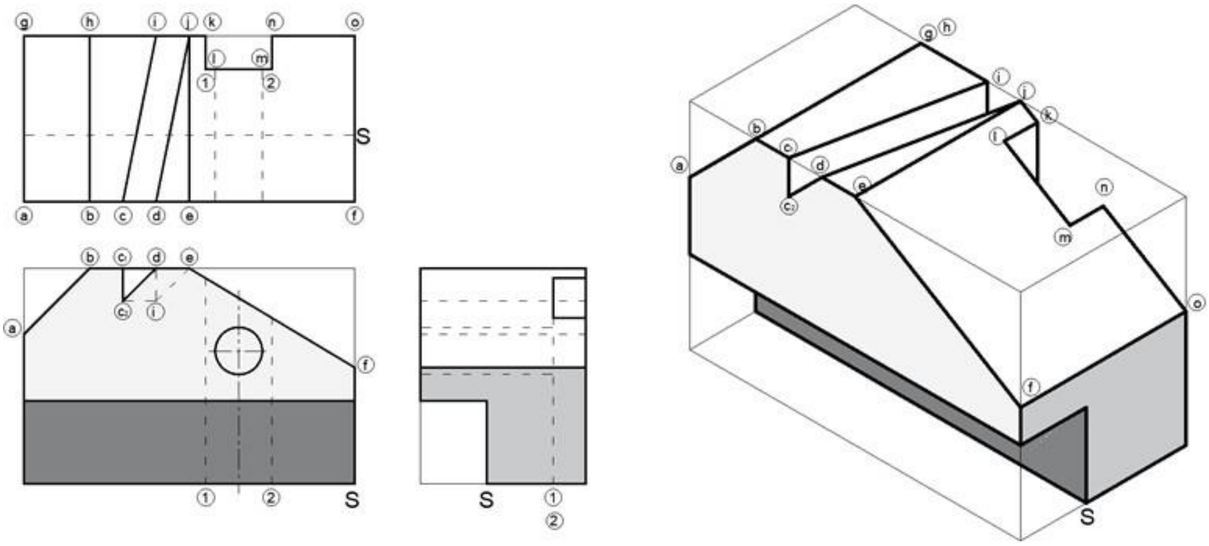

Figure 3: Orthographic and isometric views

The three van Hiele levels of geometric reasoning are used in EGD reasoning in the following manner when converting multiple 2D views of an object to one single 3D view. The cognitive descriptors of Table 1 appear in brackets where they apply: **VH2: Analysis**

Analysis of the given graphical data by identifying the orthographic net-shapes (Cognitive descriptor 1) according to their governing properties (Cognitive descriptor 2). Note: True net-shapes can be identified from the three orthographic multiviews in Figure 2, taking cognisance that slanted nets are not true shapes. At this level, reasoning is mainly inductive (Cognitive descriptor 3) due to the heuristic behaviour of “rule-inference”.

VH3: Informal deduction
In order to differentiate the different net-shapes from each other, EGD practitioners must recognise the importance of properties, and the relationships between them (Cognitive descriptor 4). They must be able to recognise a square as also being a rectangle by definition. By understanding how orthographic views relate to one another and an associated isometric view, they have to order geometric properties and connect them deductively through logical arguments (Cognitive descriptor 5). EGD practitioners must distinguish between necessary and sufficient conditions for a concept. They can form meaningful definitions and give informal arguments to justify their reasoning (Cognitive descriptor 6). This reasoning is essential to distinguish isometric and non-isometric lines from the given orthographic views as such differences require different techniques for creating isometric drawings.

VH4: Formal deduction
EGD practitioners should grasp the significance of deduction. They reason formally within the context of a mathematical system (the axiomatic rules of orthographic and isometric projection), complete with undefined terms, axioms, and underlying logical systems with definitions and theorems (Cognitive descriptor 7). In order to find the coordinates of slanted lines in the isometric view, orthographic properties can be structured to derive further information from given data by using logic more than intuition (Cognitive descriptor 8). When determining the position and extent of details that are obscured (the groove at the back), EGD

practitioners have to manipulate the intrinsic characteristics of relations to find the extents of the groove geometry purely through mental manipulations (Cognitive descriptor 9).

In summary

By virtue of the nature of EGD content, drawing tasks are traditionally quantitatively assessed at the hand of rubrics and are sometimes accompanied by short notes from the assessor. Rubric scores may well point to areas of poor performance but do not provide comprehensive information on the acquisition or non-acquisition of essential levels of cognitive reasoning.

Application of the van Hiele model across the world has been instrumental in ameliorating reasoning deficiencies in geometry by way of the model's dualistic utility. First, in a diagnostic sense, it measures reasoning on a hierarchy of cognitive competence and second, it provides learning phases for learners to cycle through content until the required cognitive skills for a particular level are acquired.

The international research on spatial reasoning interventions points to the broader applicability of structured developmental frameworks across diverse educational contexts. Research from Australia (Lowrie et al., 2018), the United States (Atit et al., 2018; Atit & Rocha, 2021; Nabutola et al., 2018), and other nations demonstrates consistent relationships between spatial reasoning and STEM performance, suggesting that fundamental spatial-mathematical connections transcend cultural and curricular differences. Lowrie's large-scale classroom interventions in Australian primary and secondary schools, involving over 800 students across multiple institutions, demonstrated that spatial training embedded within pedagogical frameworks produces substantial gains that transfer to mathematics performance across geometry, measurement, and number domains. Similarly, multi-institutional research in the United States spanning over 800 engineering students across four universities established that students hold misconceptions about fundamental engineering graphics concepts, with identifiable conceptual gaps (Nabutola et al., 2018). The consistency of findings across international contexts—regarding both the importance of spatial reasoning for STEM learning and the prevalence of spatial misconceptions—suggests that the adapted van Hiele model may have broad international applicability beyond the South African context in which this study is situated.

Rubric-based assessments tend to be characterised by a limited diagnostic capacity. In the context of EGD, we propose that adopting the van Hiele model offers the potential to not just assess VSR tasks at face value. The model provides a system that allows the pinpointing of the students' current levels of geometric reasoning and gives the lecturer a clear picture of what sort of intervention is necessary to mentor and teach the students through structured phases of cognitive development. Performance gaps are thus brought to light and transferable reasoning skills can then be taught in a targeted, need-directed way.

The three detailed examples demonstrate the framework's applicability across diverse EGD content areas, from basic orthographic-isometric conversion to complex assembly drawing interpretation. Each example shows how the nine cognitive descriptors provide specific guidance for understanding student thinking and designing appropriate interventions.

Conclusion

This paper addresses a critical gap in Engineering Graphics and Design education by adapting the van Hiele model of geometric reasoning for systematic development of visuospatial reasoning skills. Our analysis demonstrates strong alignment between van Hiele cognitive descriptors and EGD reasoning requirements, suggesting this framework can provide both diagnostic capabilities and instructional guidance. Thus, the framework's systematic approach to reasoning development offers several advantages over current EGD instructional practices: it facilitates diagnosis of student reasoning difficulties during the structured progression from basic analysis to formal deduction, simultaneously integrating spatial, analytical, and conventional knowledge.

As mentioned in the background of this article, numerous instances of poor conceptual understanding have been identified in the analysis of Grade 12 final examinations. We agree with van Hiele's sentiment that inadequate learning can often be attributed to ineffective instructional strategies and poor structured reasoning development. In answering our research question, which specifically addressed the issue of using an existing model for the teaching and learning of VSR in EGD, we looked at the core of the problem, which, according to Hurrel (2021), is predicated on the overuse of procedural knowledge in lieu of a deeper conceptual understanding. This problem is exacerbated by the fact that no model for developing such conceptual understanding and reasoning exists in the EGD field. However, the van Hiele model specifically provides a structure for the development of cognition in terms of geometry, which we link directly to VSR as required in EGD.

Our particularisation process shows a strong relationship between the cognitive descriptors of van Hiele and reasoning with EGD content. We posit that the van Hiele model of geometric reasoning is eminently suitable for EGD and can be used effectively in the teaching and learning of EGD by following the hierarchical cognitive reasoning process described in the model. Judging from the cognitive descriptors in Table 1, the constructs of visuospatial reasoning, analytical reasoning, and subject conventions can be successfully incorporated into a tailormade pedagogical framework for EGD, as demonstrated in the example we used.

We propose that by using the cognitive descriptors derived from the van Hiele model, the degree of cognitive acquisition, as suggested by quantitative scores, can be qualitatively analysed to serve as a diagnostic tool. The hierarchical levels of different types of reasoning provide specific categories of cognitive skills that have to be acquired by EGD practitioners. When a singular drawing task is analysed against the cognitive descriptors, the different cognitive activities for that task can be assigned with appropriate van Hiele levels and grouped under visuospatial reasoning, analytical reasoning, and convention knowledge. Cognitive descriptors for each van Hiele level represent different cognitive skills, yet it should be made clear that certain skills could straddle two or more levels concurrently by virtue of their uniqueness and the task requirements.

Previous studies have shown that cognitive skills are not necessarily acquired on a linear path, and skills on higher levels may be acquired while skills on lower levels may still be lacking (Patkin & Barkai, 2014). Once the task is fully described by valid cognitive descriptors spanning different van Hiele levels, the quantitative score for that item serves to inform which cognitive skills were acquired and which ones had not yet been acquired. The framework has the potential to assess conceptual deficiencies in both teachers and learners and can be used to inform remedial action plans. Teachers should understand their own and their learners' cognitive processes in terms of appropriate learning theories across hierarchical levels of acquisition. Once a complete diagnosis of reasoning deficiencies has been made, special intervention programmes and day-to-day classroom instruction can be designed around the five learning phases.

Nevertheless, this conceptual framework requires empirical validation before widespread implementation. Future research should focus on controlled studies testing the framework's effectiveness, development of reliable diagnostic instruments, teacher professional development in program design, and how cultural and contextual adaptation can be facilitated. The framework's potential extends beyond EGD to other technical drawing and design subjects, suggesting broader applications in STEM education. By providing systematic approaches to spatial reasoning development, this adapted van Hiele model could contribute to improved student retention and success in engineering and related fields.

Implications for teacher preparation and professional development

Given evidence that teachers' spatial abilities influence their pedagogical approaches (Atit & Rocha, 2021) and that spatial skills vary substantially across teacher populations (Atit et al., 2018), teacher preparation programs should explicitly address spatial reasoning development.

For Engineering Graphics and Design specifically, teacher education should address the development of teachers' own spatial reasoning and as well as training them in spatial pedagogy. The adapted van Hiele model proposed in this study provides a framework that can structure both pre-service and in-service teacher education. By understanding the developmental levels through which students' geometric reasoning progresses, teachers can more effectively diagnose students' current conceptual understanding, identify persistent misconceptions, and design instruction appropriately sequenced to support advancement through reasoning levels. Awareness of common misconceptions (Nabutola et al., 2018) can prepare teachers to anticipate and address specific conceptual difficulties before they are embedded.

The adaptation of the van Hiele model for EGD represents an important contribution toward evidence-based spatial reasoning instruction. While further empirical research is needed to validate the model's effectiveness in classroom settings, this conceptual framework provides a preliminary foundation for developing more structured approaches to spatial instruction in Engineering Graphics and Design. Future research should examine how the adapted model performs in actual classroom implementations, investigate its effectiveness across diverse student populations, and refine the cognitive descriptors based on empirical evidence of student progression through reasoning levels.

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